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# Nilpotent chiral superfield in $\mathcal{N} = 2$ supergravity and partial rigid supersymmetry breaking

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Abstract: In the framework of $\mathcal{N} = 2$ conformal supergravity in four dimensions, we introduce a nilpotent chiral superfield suitable for the description of partial supersymmetry breaking in maximally supersymmetric spacetimes. As an application, we construct Maxwell-Goldstone multiplet actions for partial $\mathcal{N} = 2 \to \mathcal{N} = 1$ supersymmetry breaking on $\mathbb{R} \times S^3$, $\mathrm{AdS}_3 \times S^1$ (or its covering $\mathrm{AdS}_3 \times \mathbb{R}$), and a pp-wave spacetime. In each of these cases, the action coincides with a unique curved-superspace extension of the $\mathcal{N} = 1$ supersymmetric Born-Infeld action, which is singled out by the requirement of $\mathsf{U}(1)$ duality invariance.

Keywords: Extended Supersymmetry, Superspaces, Supersymmetry and Duality, Supersymmetry Breaking

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## Contents



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## 1 Introduction

Inspired by the work of Antoniadis, Partouche and Taylor [1], Bagger and Galperin [2] constructed the Goldstone-Maxwell multiplet model for partially broken $\mathcal{N} = 2$ Poincaré supersymmetry in four spacetime dimensions (4D). Their model proved to coincide with the $\mathcal{N} = 1$ supersymmetric Born-Infeld action [3, 4]. Two years later, Roček and Tseytlin [5] re-derived the model of [2] using $\mathcal{N} = 2$ superfields, building on the earlier formulation due to Roček [6] for the Volkov-Akulov Goldstino model [7, 8] in terms of a nilpotent $\mathcal{N} = 1$ chiral superfield.[1]

The $\mathcal{N} = 2$ Minkowski superspace is one of many maximally supersymmetric backgrounds in 4D $\mathcal{N} = 2$ off-shell supergravity. Such superspaces were classified in [11] building on the earlier analysis [12] of maximally supersymmetric backgrounds in 5D $\mathcal{N} = 1$ off-shell supergravity. The construction in [5] is down-to-earth in the sense that it is specifically designed to describe the partial breaking of $\mathcal{N} = 2$ Poincaré supersymmetry. Here we present

[1] The same nilpotent chiral superfield was independently introduced, a few months later, by Ivanov and Kapustnikov [9] as a simple application of the general relationship between linear and nonlinear realisations of supersymmetry established in their earlier work [10].

a theoretical scheme which is suitable for the description of partial supersymmetry breaking in curved maximally supersymmetric backgrounds in 4D $\mathcal{N}=2$ off-shell supergravity. As an application of this scheme, we construct Maxwell-Goldstone multiplet actions for partial $\mathcal{N}=2 \to \mathcal{N}=1$ supersymmetry breaking on $\mathbb{R}\times S^3$, $\mathrm{AdS}_3\times S^1$ (or its covering $\mathrm{AdS}_3\times\mathbb{R}$), and a pp-wave.

This paper is organised as follows. In section 2 we introduce a nilpotent chiral superfield coupled to $\mathcal{N}=2$ conformal supergravity. In section 3 we explain how such a superfield can be used to construct a model for partially broken supersymmetry for certain maximally supersymmetric backgrounds of $\mathcal{N}=2$ supergravity. The formalism developed is applied in section 4 to re-derive the Roček-Tseytlin construction. In section 5 we construct Maxwell-Goldstone multiplet actions for partial $\mathcal{N}=2\to\mathcal{N}=1$ supersymmetry breaking on $\mathbb{R}\times S^3$, $\mathrm{AdS}_3\times S^1$ (or its covering $\mathrm{AdS}_3\times\mathbb{R}$), and a pp-wave. Concluding comments are given in section 6. The main body of the paper is accompanied by three technical appendices. In appendices A and B, we present group-theoretic formulations for four-dimensional $\mathcal{N}=1$ and $\mathcal{N}=2$ superspaces over $\mathsf{U}(2)=(S^1\times S^3)/\mathbb{Z}_2$. The maximally $\mathcal{N}=2$ supersymmetric background over $\mathbb{R}\times S^3$, which is used in section 5, is the universal covering space of the $\mathcal{N}=2$ superspace over $(S^1\times S^3)/\mathbb{Z}_2$. Appendix A also contains the group-theoretic description of $\mathcal{N}=1$ superspace over $\mathsf{U}(1,1)=(\mathrm{AdS}_3\times S^1)/\mathbb{Z}_2$. Appendix C is devoted to the discussion of a unique feature of the anti-de Sitter supersymmetry that distinguishes $\mathrm{AdS}_4$ from the other maximally supersymmetric four-dimensional backgrounds.

## 2 Nilpotent chiral superfield in $\mathcal{N}=2$ supergravity

In the framework of four-dimensional $\mathcal{N}=2$ conformal supergravity[2] we introduce a nilpotent chiral superfield constrained by

$$\bar{\mathcal{D}}^i_{\dot\alpha}\mathcal{Z} = 0\,, \tag{2.1a}$$
$$\big(\mathcal{D}^{ij}+4S^{ij}\big)\mathcal{Z}-\big(\bar{\mathcal{D}}^{ij}+4\bar{S}^{ij}\big)\bar{\mathcal{Z}} = 4\mathrm{i}\,G^{ij}\,, \tag{2.1b}$$
$$\mathcal{Z}^2 = 0\,, \tag{2.1c}$$

where $G^{ij}$ is a linear multiplet constrained by $G^{ij}G_{ij}\neq 0$. One may interpret $G^{ij}$ as the field strength of a tensor multiplet. The constraints (2.1a)–(2.1c) are invariant under the $\mathcal{N}=2$ super-Weyl transformations [13, 14] if $\mathcal{Z}$ is considered to be a primary superfield of dimension 1.

A chiral superfield constrained by (2.1b) was considered in [15] in the context of the dilaton effective action in $\mathcal{N}=2$ supergravity. In the super-Poincaré case, chiral superfields obeying the constraint (2.1b) with a constant $G^{ij}$ naturally originate in the framework of partial $\mathcal{N}=2\to\mathcal{N}=1$ supersymmetry breaking [1, 16, 17].

[2]In this paper, we use Howe's superspace formulation [13] for $\mathcal{N}=2$ conformal supergravity and follow the supergravity notation and conventions of [14]. In particular, the superspace covariant derivatives are denoted $\mathcal{D}_{\mathcal{A}}=(\mathcal{D}_a,\mathcal{D}^i_\alpha,\bar{\mathcal{D}}^{\dot\alpha}_i)$. We make use of the second-order differential operators $\mathcal{D}^{ij}:=\mathcal{D}^{\alpha(i}\mathcal{D}^{j)}_\alpha$, $\bar{\mathcal{D}}^{ij}:=\bar{\mathcal{D}}^{(i}_{\dot\alpha}\bar{\mathcal{D}}^{\dot\alpha j)}$. The $\mathsf{SU}(2)$ triplet $S^{ij}=S^{ji}$ and its conjugate $\bar{S}_{ij}=\overline{S^{ij}}$ stand for certain components of the superspace torsion tensor.

We recall that the $\mathcal{N}=2$ tensor multiplet is described in curved superspace by its gauge invariant field strength $G^{ij}$ which is a linear multiplet. The latter is defined to be a real $\mathsf{SU}(2)$ triplet (that is, $G^{ij}=G^{ji}$ and $\bar{G}_{ij}:=\overline{G^{ij}}=G_{ij}$) subject to the covariant constraints [18–20]

$$\mathcal{D}_\alpha^{(i}G^{jk)}=\bar{\mathcal{D}}_{\dot\alpha}^{(i}G^{jk)}=0\ . \tag{2.2}$$

These constraints are solved in terms of a chiral prepotential $\Psi$ [21–24] via

$$G^{ij}=\frac{1}{4}\Big(\mathcal{D}^{ij}+4S^{ij}\Big)\Psi+\frac{1}{4}\Big(\bar{\mathcal{D}}^{ij}+4\bar{S}^{ij}\Big)\bar{\Psi}\,,\qquad \bar{\mathcal{D}}^i_{\dot\alpha}\Psi=0\,, \tag{2.3}$$

which is invariant under Abelian gauge transformations

$$\delta_\Lambda\Psi=\mathrm{i}\Lambda\,, \tag{2.4}$$

with $\Lambda$ a reduced chiral superfield,

$$\bar{\mathcal{D}}^i_{\dot\alpha}\Lambda=0\,, \tag{2.5a}$$

$$\big(\mathcal{D}^{ij}+4S^{ij}\big)\Lambda-\big(\bar{\mathcal{D}}^{ij}+4\bar{S}^{ij}\big)\bar{\Lambda}=0\ . \tag{2.5b}$$

We recall that the field strength of an Abelian vector multiplet is a reduced chiral superfield [25].

The constraints on $\Lambda$ can be solved in terms of the Mezincescu prepotential [26] (see also [21]), $U_{ij}=U_{ji}$, which is an unconstrained real $\mathsf{SU}(2)$ triplet. The curved-superspace solution is [27]

$$\Lambda=\frac{1}{4}\bar{\Delta}\Big(\mathcal{D}^{ij}+4S^{ij}\Big)U_{ij}\ . \tag{2.6}$$

Here $\bar{\Delta}$ denotes the chiral projection operator [28, 29]

$$\begin{aligned}\bar{\Delta}&=\frac{1}{96}\Big((\bar{\mathcal{D}}^{ij}+16\bar{S}^{ij})\bar{\mathcal{D}}_{ij}-(\bar{\mathcal{D}}^{\dot\alpha\dot\beta}-16\bar{Y}^{\dot\alpha\dot\beta})\bar{\mathcal{D}}_{\dot\alpha\dot\beta}\Big)\\&=\frac{1}{96}\Big(\bar{\mathcal{D}}_{ij}(\bar{\mathcal{D}}^{ij}+16\bar{S}^{ij})-\bar{\mathcal{D}}_{\dot\alpha\dot\beta}(\bar{\mathcal{D}}^{\dot\alpha\dot\beta}-16\bar{Y}^{\dot\alpha\dot\beta})\Big)\,,\end{aligned} \tag{2.7}$$

with $\bar{\mathcal{D}}^{\dot\alpha\dot\beta}:=\bar{\mathcal{D}}^{(\dot\alpha}_k\bar{\mathcal{D}}^{\dot\beta)k}$. Its main properties can be formulated using a super-Weyl inert scalar $V$. It holds that

$$\bar{\mathcal{D}}^{\dot\alpha}_i\bar{\Delta}V=0\,, \tag{2.8a}$$

$$\delta_\sigma V=0\quad\Longrightarrow\quad\delta_\sigma\bar{\Delta}V=2\sigma\bar{\Delta}V\,, \tag{2.8b}$$

$$\int\mathrm{d}^4x\,\mathrm{d}^4\theta\,\mathrm{d}^4\bar\theta\,E\,V=\int\mathrm{d}^4x\,\mathrm{d}^4\theta\,\mathcal{E}\,\bar{\Delta}V\,, \tag{2.8c}$$

where the real unconstrained parameter $\sigma$ corresponds to the super-Weyl transformations [14].[3] Here $E$ and $\mathcal{E}$ denote the full superspace and chiral densities, respectively.

The constraints (2.1a) and (2.1b) define a deformed reduced chiral superfield. These constraints may be re-cast in the language of superforms as $\mathrm{d}F=H$, where $F$ is a two-form and $H$ is the three-form field strength, $\mathrm{d}H=0$, describing the tensor multiplet [29], see

[3] The parameter $\sigma$ was denoted $2U$ in [14].

also [30].[4] Switching $H$ off, $H = 0$, turns $F$ into the two-form field strength of the vector multiplet.

The constraint (2.1b) naturally originates as follows. Consider the model for a massive improved tensor multiplet coupled to $\mathcal{N} = 2$ conformal supergravity [31, 32]. The action of this model in the form given in [27] is

$$S_{\text{tensor}} = -\int \mathrm{d}^4x\, \mathrm{d}^4\theta\, \mathcal{E} \left\{ \Psi \mathbb{W} + \frac{1}{4}\mu(\mu + \mathrm{i}e)\Psi^2 \right\} + \text{c.c.}\,, \tag{2.9}$$

where $\mu$ and $e$ are real parameters, with $\mu \neq 0$ (the tensor multiplet mass can be shown to be $m = \sqrt{\mu^2 + e^2}$). The kinetic term involves the composite [33]

$$\mathbb{W} := -\frac{G}{8}(\bar{\mathcal{D}}_{ij} + 4\bar{S}_{ij}) \left( \frac{G^{ij}}{G^2} \right)\,, \tag{2.10}$$

which proves to be a reduced chiral superfield.[5] For $m = 0$ the above action describes the improved tensor multiplet [33]. We introduce a Stückelberg-type extension of the model

$$\widetilde{S}_{\text{tensor}} = -\int \mathrm{d}^4x\, \mathrm{d}^4\theta\, \mathcal{E} \left\{ \Psi \mathbb{W} + \frac{1}{4}\mu(\mu + \mathrm{i}e)(\Psi - \mathrm{i}W)^2 \right\} + \text{c.c.}\,, \tag{2.11}$$

where $W$ is the field strength of a vector multiplet. The action is invariant under the gauge transformation (2.4) accompanied by

$$\delta_\Lambda W = \Lambda\,. \tag{2.12}$$

The original action (2.9) is obtained from (2.11) by choosing a gauge $W = 0$. Now one can see that the superfield $\mathcal{Z} := W + \mathrm{i}\Psi$ obeys the constraint (2.1b).

It is well known that the functional

$$\mathrm{i} \int \mathrm{d}^4x\, \mathrm{d}^4\theta\, \mathcal{E}\, W^2 + \text{c.c.} \tag{2.13}$$

is a total derivative. Since the mass term in (2.11) is invariant under the gauge transformation (2.4) and (2.12), it follows that, given a chiral superfield $\mathcal{Z}$ constrained by (2.1b), the functional

$$I = \int \mathrm{d}^4x\, \mathrm{d}^4\theta\, \mathcal{E} \left\{ \mathcal{Z}\Psi - \frac{\mathrm{i}}{2}\Psi^2 \right\} + \text{c.c.} \tag{2.14}$$

is invariant under the gauge transformation (2.4), $\delta_\Lambda I = 0$.

The constraints (2.1a)–(2.1c) imply that, for certain supergravity backgrounds, the degrees of freedom described by the $\mathcal{N} = 2$ chiral superfield $\mathcal{Z}$ are in a one-to-one correspondence with those of an Abelian $\mathcal{N} = 1$ vector multiplet. The specific feature of such $\mathcal{N} = 2$ supergrvaity backgrounds is that they possess an $\mathcal{N} = 1$ subspace of the full $\mathcal{N} = 2$ superspace. This property is not universal. In particular, there exist maximally $\mathcal{N} = 2$ supersymmetric backgrounds with no admissible truncation to $\mathcal{N} = 1$ [11].

[4] We are grateful to Joseph Novak for this observation.

[5] The superfield (2.10) is one of the simplest applications of the powerful approach to generate composite reduced chiral multiplets which was presented in [27].

## 3 Maximally $\mathcal{N} = 2$ supersymmetric backgrounds and partial supersymmetry breaking

So far we have discussed an arbitrary supergravity background. Now we restrict our consideration to a maximally supersymmetric background $\mathbb{M}^{4|8}$ with the property that the chiral prepotential $\Psi$ for $G^{ij}$ may be chosen such that the following two conditions hold. Firstly, the complex linear multiplet

$$G_{+}^{ij} := \frac{1}{4}\Big(\mathcal{D}^{ij} + 4S^{ij}\Big)\Psi \tag{3.1}$$

is covariantly constant and null,

$$\mathcal{D}_{\mathcal{A}} G_{+}^{ij} = 0\,, \tag{3.2}$$

$$G_{+}^{ij} G_{+ij} = 0\;. \tag{3.3}$$

Secondly, the prepotential $\Psi$ may be chosen to be nilpotent,

$$\Psi^2 = 0\;. \tag{3.4}$$

The null condition for $G_{+}^{ij}$ means that $G_{+}^{ij} = q^i q^j$, for some isospinor $q^i$. It follows that $G^{ij} = G_{+}^{ij} + G_{-}^{ij}$ is covariantly constant,

$$\mathcal{D}_{\mathcal{A}} G^{ij} = 0\,, \tag{3.5}$$

where we have denoted $G_{-}^{ij} := \frac{1}{4}\Big(\bar{\mathcal{D}}^{ij} + 4\bar{S}^{ij}\Big)\bar{\Psi}$.

We are going to show that the following functional

$$I = \int \mathrm{d}^4x\, \mathrm{d}^4\theta\, \mathcal{E}\, \Psi \mathcal{Z} \tag{3.6}$$

is supersymmetric. Here $\mathcal{Z}$ is the nilpotent chiral superfield (2.1), which is assumed to be a composite of the dynamical fields. The complex linear multiplet (3.1) and its chiral prepotential $\Psi$ are background fields associated with the background superspace $\mathbb{M}^{4|8}$. Since the covariant derivatives $\mathcal{D}_{\mathcal{A}}$ are invariant under the isometry transformations of $\mathbb{M}^{4|8}$, the fields $G_{+}^{ij}$ and $\Psi$ do not change under such transformations. Let $\xi$ be a Killing supervector field for $\mathbb{M}^{4|8}$ (see section 6.4 of [34] and [35] for general discussions). Then

$$\delta_\xi I = \int \mathrm{d}^4x\, \mathrm{d}^4\theta\, \mathcal{E}\, \Psi \delta_\xi \mathcal{Z} = -\int \mathrm{d}^4x\, \mathrm{d}^4\theta\, \mathcal{E}\, \mathcal{Z} \delta_\xi \Psi\;. \tag{3.7}$$

We introduce a reduced chiral superfield $W$ by

$$\mathcal{Z} = W + \mathrm{i}\Psi\,, \qquad W = \frac{1}{4}\bar{\Delta}\Big(\mathcal{D}^{ij} + 4S^{ij}\Big)U_{ij}\,, \tag{3.8}$$

where $U_{ij}$ is the Mezincescu prepotential for the reduced chiral superfield $W$. Since $\Psi\delta_\xi\Psi = 0$, we have

$$\delta_\xi I = -\int \mathrm{d}^4x\, \mathrm{d}^4\theta\, \mathcal{E}\, \mathcal{Z}\delta_\xi \Psi = -\int \mathrm{d}^4x\, \mathrm{d}^4\theta\, \mathcal{E}\, W \delta_\xi \Psi$$

$$
\begin{aligned}
&= -\frac{1}{4}\int \mathrm{d}^4x\, \mathrm{d}^4\theta\, \mathrm{d}^4\bar\theta\, E\, U_{ij}\Big(\mathcal{D}^{ij} + 4S^{ij}\Big)\delta_\xi \Psi \\
&= -\frac{1}{4}\int \mathrm{d}^4x\, \mathrm{d}^4\theta\, \mathrm{d}^4\bar\theta\, E\, U_{ij}\delta_\xi\Big(\mathcal{D}^{ij} + 4S^{ij}\Big)\Psi \\
&= -\int \mathrm{d}^4x\, \mathrm{d}^4\theta\, \mathrm{d}^4\bar\theta\, E\, U_{ij}\delta_\xi G_+^{ij} = 0\ .
\end{aligned}
\tag{3.9}
$$

In the next two sections, it will be shown that the action

$$
S = -\frac{\mathrm{i}}{4}\int \mathrm{d}^4x\, \mathrm{d}^4\theta\, \mathcal{E}\, \Psi \mathcal{Z} + \text{c.c.} \tag{3.10}
$$

describes the Maxwell-Goldstone multiplet for partial $\mathcal{N} = 2 \to \mathcal{N} = 1$ supersymmetry breaking on the maximally supersymmetric backgrounds specified.

The above derivation does not use the null condition (3.3). The latter is introduced for the $\mathcal{N} = 2$ superspace $\mathbb{M}^{4|8}$ to possess an $\mathcal{N} = 1$ subspace.

## 4 Example: the super-Poincaré case

The simplest maximally supersymmetric background is $\mathcal{N} = 2$ Minkowski superspace. In this superspace, every constant real $\mathsf{SU}(2)$ triplet $G^{ij}$ is covariantly constant,

$$
D_\mathcal{A} G^{ij} = 0\,, \tag{4.1}
$$

where $D_\mathcal{A} = (\partial_a, D_\alpha^i, \bar D_i^{\dot\alpha})$ are the flat superspace covariant derivatives. Let $\Psi$ be a chiral prepotential for $G^{ij}$, $\bar D^i_{\dot\alpha}\Psi = 0$. We represent

$$
G^{ij} = G_+^{ij} + G_-^{ij}\,, \qquad G_+^{ij} = \frac{1}{4} D^{ij}\Psi\,, \qquad G_-^{ij} = \frac{1}{4}\bar D^{ij}\bar\Psi\ . \tag{4.2}
$$

It is always possible to choose the prepotential $\Psi$ such that the following properties hold:

$$
\Psi^2 = 0\,, \qquad D_\mathcal{A} G_+^{ij} = 0\,, \qquad G_+^{ij} G_{+ij} = 0\ . \tag{4.3}
$$

In $\mathcal{N} = 2$ Minkowski superspace, the constraints (2.1a)–(2.1c) turn into

$$
\bar D^i_{\dot\alpha}\mathcal{Z} = 0\,, \tag{4.4a}
$$

$$
D^{ij}\mathcal{Z} - \bar D^{ij}\bar{\mathcal{Z}} = 4\mathrm{i}\, G^{ij}\,, \tag{4.4b}
$$

$$
\mathcal{Z}^2 = 0\ . \tag{4.4c}
$$

The action (3.10) becomes

$$
S = -\frac{\mathrm{i}}{4}\int \mathrm{d}^4x\, \mathrm{d}^4\theta\, \mathcal{Z}\Psi \ + \ \text{c.c.} \tag{4.5}
$$

Since $G_+^{ij}$ is constant, it is invariant under the $\mathcal{N} = 2$ supersymmetry transformations. In accordance with the analysis given in the previous section, the action is $\mathcal{N} = 2$ supersymmetric.

For the Grassmann coordinates $\theta_i^\alpha$ and $\bar\theta^j_{\dot\alpha}$ of $\mathcal{N}=2$ Minkowski superspace, as well as for the spinor covariant derivatives $D^i_\alpha$ and $\bar D_i^{\dot\alpha}$, it is useful to label the values of their $R$-symmetry indices as $i,j=\underline{1},\underline{2}$. Without loss of generality we can choose

$$G_+^{ij} = -\mathrm{i}\delta^i_{\underline{2}}\delta^j_{\underline{2}}\,, \qquad \Psi = \mathrm{i}\theta^\alpha_{\underline{2}}\theta_{\alpha\underline{2}}\ . \tag{4.6}$$

We can now reproduce the results of [2] from the $\mathcal{N}=2$ setup described. In order to solve the constraints (4.4), it is useful to carry out a reduction to $\mathcal{N}=1$ Minkowski superspace.

Given a superfield $U(x,\theta_i,\bar\theta^i)$ on $\mathcal{N}=2$ Minkowski superspace, we introduce its bar-projection

$$U| := U(x,\theta_i,\bar\theta^i)|_{\theta_{\underline{2}}=\bar\theta^{\underline{2}}=0}\,, \tag{4.7}$$

which is a superfield on $\mathcal{N}=1$ Minkowski superspace with the Grassmann coordinates $\theta^\alpha = \theta^\alpha_{\underline{1}}$ and $\bar\theta_{\dot\alpha} = \bar\theta^{\underline{1}}_{\dot\alpha}$ and the spinor covariant derivatives $D_\alpha = D^{\underline{1}}_\alpha$ and $\bar D^{\dot\alpha} = \bar D^{\dot\alpha}_{\underline{1}}$. The background superfield $\Psi$ is characterised by the properties

$$\Psi| = 0\,, \qquad D^{\underline{2}}_\alpha\Psi| = 0\ . \tag{4.8}$$

Since $\mathcal{Z}^2=0$, the constraints (4.4) imply

$$(D^{\alpha\underline{2}}\mathcal{Z})D^{\underline{2}}_\alpha\mathcal{Z} + \mathcal{Z}\bar D_{\underline{11}}\bar{\mathcal{Z}} + 4\mathcal{Z} = 0\ . \tag{4.9}$$

Taking the bar-projection of this constraint gives

$$X + \frac{1}{4}X\bar D^2\bar X = W^2\,, \qquad W^2 := W^\alpha W_\alpha\,, \tag{4.10}$$

where we have introduced the $\mathcal{N}=1$ components of $\mathcal{Z}$:

$$X := \mathcal{Z}|\,, \qquad W_\alpha := -\frac{\mathrm{i}}{2}D^{\underline{2}}_\alpha\mathcal{Z}|\ . \tag{4.11a}$$

These superfields satisfy the constraints

$$\bar D_{\dot\alpha}X = 0\,, \qquad \bar D_{\dot\alpha}W_\alpha = 0\,, \qquad D^\alpha W_\alpha = \bar D_{\dot\alpha}\bar W^{\dot\alpha}\ . \tag{4.11b}$$

The constraints on $W_\alpha$ tell us that it can be interpreted as the field strength of an Abelian $\mathcal{N}=1$ vector multiplet. The constraint (4.10) is equivalent to the Bagger-Galperin constraint [2]. Its general solution is

$$X = W^2 - \frac{1}{2}\bar D^2\frac{W^2\bar W^2}{\left(1+\frac{1}{2}A+\sqrt{1+A+\frac{1}{4}B^2}\right)}\,, \tag{4.12a}$$

$$A = \frac{1}{2}\big(D^2W^2 + \bar D^2\bar W^2\big)\,, \qquad B = \frac{1}{2}\big(D^2W^2 - \bar D^2\bar W^2\big)\ . \tag{4.12b}$$

Upon reduction to $\mathcal{N}=1$ superspace, the action (4.5) becomes

$$I = \frac{1}{4}\int \mathrm{d}^4x\,\mathrm{d}^2\theta\,X + \frac{1}{4}\int\mathrm{d}^4x\,\mathrm{d}^2\bar\theta\,\bar X\ . \tag{4.13}$$

This is the $\mathcal{N}=1$ supersymmetric Born-Infeld action. Being manifestly $\mathcal{N}=1$ supersymmetric, the action is also invariant under the second nonlinearly realised supersymmetry transformation [2]

$$\delta_\epsilon W_\alpha = \epsilon_\alpha + \frac{1}{4}\epsilon_\alpha \bar{D}^2\bar{X} + \mathrm{i}\bar{\epsilon}^{\dot\beta}\partial_{\alpha\dot\beta}X \quad \Longrightarrow \quad \delta_\epsilon X = 2\epsilon^\alpha W_\alpha \ . \tag{4.14}$$

For completeness, we re-derive this result.

Let $U$ be a scalar superfield on $\mathcal{N}=2$ Minkowski superspace. Its isometry transformation is

$$\delta_\xi U = -\xi U \,, \tag{4.15}$$

where

$$\xi = \bar{\xi} = \xi^{\mathcal{A}} D_{\mathcal{A}} = \xi^a\partial_a + \xi^\alpha_i D^i_\alpha + \bar{\xi}^i_{\dot\alpha}\bar{D}^{\dot\alpha}_i \tag{4.16}$$

is a Killing supervector field of Minkowski superspace,[6]

$$\xi^\alpha_i = -\frac{\mathrm{i}}{8}\bar{D}_{\dot\beta i}\xi^{\dot\beta\alpha}\,, \qquad D^i_{(\alpha}\xi_{\beta)\dot\beta} = \bar{D}_{i(\dot\alpha}\xi_{\beta\dot\beta)} = 0\,, \qquad D^i_\alpha\xi^\alpha_i = 0 \ . \tag{4.17}$$

The Killing supervector field generating the supersymmetry transformation is characterised by the components

$$\xi^a = 2\mathrm{i}(\theta_i\sigma^a\bar{\epsilon}^i - \epsilon_i\sigma^a\bar{\theta}^i)\,, \qquad \xi^\alpha_i = \epsilon^\alpha_i = \text{const} \ . \tag{4.18}$$

Applying this transformation to $\mathcal{Z}$ gives $\delta_\xi\mathcal{Z} = -(\xi^a\partial_a + \xi^\alpha_i D^i_\alpha)\mathcal{Z}$. We now consider only the second supersymmetry transformation by choosing $\epsilon^\alpha_{\underline{1}} = 0$ and $\epsilon^\alpha_{\underline{2}} = \epsilon^\alpha$. It acts on the $\mathcal{N}=1$ superfields (4.11a) as follows

$$\delta_\epsilon X = \delta_\xi \mathcal{Z}| = -(\xi\mathcal{Z})| = -\epsilon^\alpha(D^{\underline{2}}_\alpha\mathcal{Z})| = -2\mathrm{i}\epsilon^\alpha W_\alpha\,, \tag{4.19a}$$

$$\delta_\epsilon W_\alpha = -\frac{\mathrm{i}}{2}(D^{\underline{2}}_\alpha\delta_\xi\mathcal{Z})| = -\mathrm{i}\epsilon_\alpha - \frac{\mathrm{i}}{4}\epsilon_\alpha\bar{D}^2\bar{X} - \bar{\epsilon}^{\dot\beta}\partial_{\alpha\dot\beta}X\,, \tag{4.19b}$$

where we have made use of the constraints obeyed by $\mathcal{Z}$ and $X$. The supersymmetry transformation (4.14) follows from (4.19) upon a rescaling of $\epsilon^\alpha$.

# 5 Maxwell-Goldstone multiplet for partially broken rigid supersymmetry in curved space

We turn to applying the theoretical framework of section 3 to maximally supersymmetric curved backgrounds in $\mathcal{N}=2$ supergravity.

## 5.1 Curved $\mathcal{N}=2$ superspace backgrounds

We consider a maximally supersymmetric background $\mathbb{M}^{4|8}$ described by the following algebra of covariant derivatives[7]

$$\{\mathcal{D}^i_\alpha, \mathcal{D}^j_\beta\} = \{\bar{\mathcal{D}}^{\dot\alpha}_i, \bar{\mathcal{D}}^{\dot\beta}_j\} = 0\,, \tag{5.1a}$$

[6]It follows from (4.17) that $\xi^\alpha_i$ is chiral, $\bar{D}^j_{\dot\beta}\xi^\alpha_i = 0$.

[7]Here $M_{ab}$, $J^{kl}$ and $Y$ are the Lorentz, $\mathsf{SU}(2)$ and $\mathsf{U}(1)$ generators, respectively, defined as in [14].

$$\{\mathcal{D}^i_\alpha, \bar{\mathcal{D}}^{\dot\beta}_j\} = -2\mathrm{i}\delta^i_j \mathcal{D}_\alpha{}^{\dot\beta} + 4\mathrm{i}G^{\gamma\dot\beta i}{}_j M_{\alpha\gamma} + 4\mathrm{i}G_{\alpha\dot\gamma}{}^i{}_j \bar{M}^{\dot\gamma\dot\beta}$$
$$-4\mathrm{i}\delta^i_j G_\alpha{}^{\dot\beta kl} J_{kl} - 2\mathrm{i}G_\alpha{}^{\dot\beta i}{}_j Y\,, \tag{5.1b}$$
$$[\mathcal{D}_a, \mathcal{D}^j_\beta] = (\tilde\sigma_a)^{\dot\alpha\gamma} G_{\beta\dot\alpha}{}^j{}_k \mathcal{D}^k_\gamma\,, \qquad [\mathcal{D}_a, \bar{\mathcal{D}}_{\dot\beta j}] = -(\tilde\sigma_a)^{\dot\gamma\alpha} G_{\alpha\dot\beta j}{}^k \bar{\mathcal{D}}_{\dot\gamma k}\,, \tag{5.1c}$$

where the torsion tensor $G^{ij}_a$ is annihilated by the spinor covariant derivatives,

$$\mathcal{D}^i_\alpha G^{jk}_b = 0\,, \qquad \bar{\mathcal{D}}^i_{\dot\alpha} G^{jk}_b = 0\,. \tag{5.1d}$$

This algebra is obtained from that corresponding to $\mathcal{N}=2$ conformal supergravity, and given by eq. (2.8) in [14], by (i) switching off the components $S^{ij}$, $Y_{\alpha\beta}$, $W_{\alpha\beta}$ and $G_{\alpha\dot\alpha}$ of the torsion tensor; and (ii) imposing (5.1d). The constraints (5.1d) are required by the theorem [12] that all fermionic components of the superspace torsion tensor must vanish in maximally supersymmetric backgrounds.

In complete analogy with the 5D case [12], the constraints (5.1d) imply the following integrability condition

$$G_a{}^{k(i} G_b{}^{j)}{}_k = 0\,. \tag{5.2}$$

As shown in [12], the general solution of the conditions (5.1d) and (5.2) is

$$G_b{}^{kl} = -\frac{1}{4} g_b s^{kl}\,, \qquad \mathcal{D}^i_\alpha g_b = 0\,, \quad \bar{\mathcal{D}}^i_{\dot\alpha} g_b = 0\,, \qquad \mathcal{D}_A s^{kl} = 0\,, \tag{5.3}$$

for some real vector $g_b$ and real $\mathsf{SU}(2)$ triplet $s^{kl}$. The latter may be normalised as

$$s^{ij} s_{ij} = 2\,. \tag{5.4}$$

Since $g^2 = g^a g_a$ is constant, $\mathcal{D}_A g^2 = 0$, there are in fact three different superspaces described by the above algebra: (i) if $g_a$ is time-like, $g^2 < 0$, the bosonic body of $\mathbb{M}^{4|8}$ is $\mathbb{R}\times S^3$; (ii) if $g_a$ is space-like, $g^2 > 0$, the bosonic body of $\mathbb{M}^{4|8}$ is $\mathrm{AdS}_3\times\mathbb{R}$; (iii) in the null case, $g^2 = 0$, the spacetime geometry is a pp-wave. We will denote these superspaces as $\mathbb{M}^{4|8}_T$, $\mathbb{M}^{4|8}_S$ and $\mathbb{M}^{4|8}_N$, respectively. These backgrounds were constructed in [11], and they have 5D cousins [12].

In order to get some more insight into the structure of the superspace geometry (5.1), a specific value of $g^2$ has to be fixed. It suffices to consider the superspace $\mathbb{M}^{4|8}_T$, since the other two cases may be treated similarly. As a supermanifold, $\mathbb{M}^{4|8}_T$ is the universal covering of the 4D $\mathcal{N}=2$ superspace introduced in appendix B.

In the case $g^2<0$, it is possible to choose a Lorentz and $\mathsf{SU}(2)_R$ gauge such that

$$g_a = (g,0,0,0)\,, \qquad s_i{}^j = -\mathrm{i}(\sigma^3)_i{}^j = \mathrm{i}(-1)^i \delta^j_i\,. \tag{5.5}$$

As shown in [11], the algebra of covariant derivatives is equivalent to

$$\{\mathcal{D}^i_\alpha, \mathcal{D}^j_\beta\} = \{\bar{\mathcal{D}}^{\dot\alpha}_i, \bar{\mathcal{D}}^{\dot\beta}_j\} = 0\,, \qquad \{\mathcal{D}^i_\alpha, \bar{\mathcal{D}}^{\dot\beta}_j\} = -2\mathrm{i}\delta^i_j (\sigma^a)_\alpha{}^{\dot\beta} \mathcal{D}^{(i)}_a\,, \tag{5.6a}$$
$$[\mathcal{D}^{(i)}_a, \mathcal{D}^j_\beta] = \frac{\mathrm{i}}{2}\delta^{ij}(-1)^j (\sigma_a)_{\beta\dot\beta}\, g^{\dot\beta\gamma} \mathcal{D}^j_\gamma\,, \tag{5.6b}$$
$$[\mathcal{D}^{(i)}_a, \mathcal{D}^{(j)}_b] = (-1)^{j+1}\delta^{ij}\varepsilon_{abc}{}^d g^c \mathcal{D}^{(j)}_d\,, \tag{5.6c}$$



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where we have introduced the "improved" vector covariant derivatives

$$\mathcal{D}_a^{(i)} := \mathcal{D}_a + \frac{1}{2} g_a s^{kl} J_{kl} + (-1)^i \left( \frac{1}{4} \varepsilon_{abcd}\, g^b M^{cd} + \mathrm{i} g_a Y \right) . \tag{5.7}$$

These (anti-)commutation relations correspond to the superalgebra $\mathfrak{su}(2|1) \times \mathfrak{su}(2|1)$.

The superspace geometry of $\mathbb{M}^{4|8}_T$ can be described, e.g., in terms of the covariant derivatives $\tilde{\mathcal{D}}_{\mathcal{A}} = (\mathcal{D}_a^{(1)}, \mathcal{D}^i_\alpha, \bar{\mathcal{D}}^{\dot\alpha}_i)$. In accordance with (5.6), the operators $(\mathcal{D}_a^{(1)}, \mathcal{D}^{\underline{1}}_\alpha, \bar{\mathcal{D}}^{\dot\alpha}_{\underline{1}})$ form a closed algebra isomorphic to that of the superalgebra $\mathfrak{su}(2|1)$. This property means that the $\mathcal{N} = 2$ superspace $\mathbb{M}^{4|8}_T$ possesses an $\mathcal{N} = 1$ subspace which will be denoted $\mathbb{M}^{4|4}_T$. It turns out that all the conditions (3.2)–(3.4) can be met in the case of $\mathbb{M}^{4|8}_T$. In particular, this superspace allows the existence of covariantly constant complex $\mathsf{SU}(2)$ triplets $G^{ij}_+$. Since the graded commutation relations for $\tilde{\mathcal{D}}_{\mathcal{A}}$ involve neither Lorentz nor $\mathsf{SU}(2)$ curvature tensors, the Lorentz and $\mathsf{SU}(2)$ connections may be gauged away. In such a gauge, every constant complex $\mathsf{SU}(2)$ triplets $G^{ij}_+$ is covariantly constant.

Since the superspaces $\mathbb{M}^{4|8}_T$, $\mathbb{M}^{4|8}_S$ and $\mathbb{M}^{4|8}_N$ meet the requirements (3.2)–(3.4), the formalism of section 3 may be used to construct a Maxwell-Goldstone multiplet action for partial supersymmetry breaking. Instead of implementing the scheme directly, we will take a shortcut to constructing such actions on the $\mathcal{N} = 1$ subspaces of the superspaces $\mathbb{M}^{4|8}_T$, $\mathbb{M}^{4|8}_S$ and $\mathbb{M}^{4|8}_N$.

### 5.2 Goldstone multiplet for partially broken supersymmetry

We consider a maximally supersymmetric background $\mathbb{M}^{4|4}$ described by the following algebra of $\mathcal{N} = 1$ covariant derivatives:

$$\{\mathcal{D}_\alpha, \mathcal{D}_\beta\} = 0\,, \qquad \{\bar{\mathcal{D}}_{\dot\alpha}, \bar{\mathcal{D}}_{\dot\beta}\} = 0\,, \qquad \{\mathcal{D}_\alpha, \bar{\mathcal{D}}_{\dot\beta}\} = -2\mathrm{i}\mathcal{D}_{\alpha\dot\beta}\,, \tag{5.8a}$$

$$[\mathcal{D}_\alpha, \mathcal{D}_{\beta\dot\beta}] = \mathrm{i}\varepsilon_{\alpha\beta} G^\gamma{}_{\dot\beta} \mathcal{D}_\gamma\,, \qquad [\bar{\mathcal{D}}_{\dot\alpha}, \mathcal{D}_{\beta\dot\beta}] = -\mathrm{i}\varepsilon_{\dot\alpha\dot\beta} G_\beta{}^{\dot\gamma} \bar{\mathcal{D}}_{\dot\gamma}\,, \tag{5.8b}$$

$$[\mathcal{D}_{\alpha\dot\alpha}, \mathcal{D}_{\beta\dot\beta}] = -\mathrm{i}\varepsilon_{\dot\alpha\dot\beta} G_\beta{}^{\dot\gamma} \mathcal{D}_{\alpha\dot\gamma} + \mathrm{i}\varepsilon_{\alpha\beta} G^\gamma{}_{\dot\beta} \mathcal{D}_{\gamma\dot\alpha}\,, \tag{5.8c}$$

where the torsion tensor $G_a$ is covariantly constant,

$$\mathcal{D}_A G_b = 0\,. \tag{5.8d}$$

This is a special case of the superspace geometry for $\mathcal{N} = 1$ old minimal supergravity [36, 37] reviewed in [34]. The above algebra is obtained from the supergravity (anti-)commutation relations (5.5.6) and (5.5.7) in [34] by (i) switching off the chiral torsion superfields $R$ and $W_{\alpha\beta\gamma}$ and their conjugates; and (ii) imposing the condition (5.8d).

Since $G^2 = G^a G_a$ is constant, the geometry (5.8) describes three different superspaces, $\mathbb{M}^{4|4}_T$, $\mathbb{M}^{4|4}_S$ and $\mathbb{M}^{4|4}_N$, which correspond to the choices $G^2 < 0$, $G^2 > 0$ and $G^2 = 0$, respectively. These $\mathcal{N} = 1$ superspaces originate as the $\mathcal{N} = 1$ subspaces of the $\mathcal{N} = 2$ superspaces of $\mathbb{M}^{4|8}_T$, $\mathbb{M}^{4|8}_S$ and $\mathbb{M}^{4|8}_N$, respectively, considered in the previous subsection.[8] We recall that the Lorentzian manifolds supported by these superspaces are $\mathbb{R} \times S^3$, $\mathrm{AdS}_3 \times$

[8]In the time-like case, $G^2 < 0$, the graded commutation relations (5.8) are obtained from (5.6) by choosing $i, j = \underline{1}$ and setting $G_a = g_a$.

$S^1$ or its covering $\mathrm{AdS}_3 \times \mathbb{R}$, and a pp-wave spacetime, respectively.[9] As a supermanifold, $\mathbb{M}^{4|4}_T$ is the universal covering of the $\mathcal{N}=1$ superspace $\mathcal{M}^{4|4}$ introduced in section A.1. The isometry group of $\mathcal{M}^{4|4}$ is $\mathsf{SU}(2|1) \times \mathsf{U}(2)$. As a supermanifold, $\mathbb{M}^{4|4}_S$ is the universal covering of the $\mathcal{N}=1$ superspace $\widetilde{\mathcal{M}}^{4|4}$ introduced in section A.2. The isometry group of $\widetilde{\mathcal{M}}^{4|4}$ is $\mathsf{SU}(1,1|1) \times \mathsf{U}(2)$.

The superspace $\mathbb{M}^{4|4}$ allows the existence of covariantly constant spinors,

$$\mathcal{D}_A \epsilon_\alpha = 0 \ . \tag{5.9}$$

Such a spinor is constant in a gauge in which the Lorentz connection vanishes.

By analogy with the flat-superspace case, we consider the following $\mathcal{N}=1$ supersymmetric theory with action

$$S = \frac{1}{4} \int \mathrm{d}^4x \, \mathrm{d}^2\theta \, \mathcal{E} \, X + \mathrm{c.c.} \, , \tag{5.10}$$

where the covariantly chiral superfield $X$ is a unique solution of the constraint

$$X + \frac{1}{4} X \bar{\mathcal{D}}^2 \bar{X} = W^2 \ . \tag{5.11}$$

The superfield $W_\alpha$ is the chiral field strength of an Abelian vector multiplet and, together with its complex conjugate $\bar{W}_{\dot\alpha}$, it obeys the Bianchi identity

$$\mathcal{D}^\alpha W_\alpha = \bar{\mathcal{D}}_{\dot\alpha} \bar{W}^{\dot\alpha} \ . \tag{5.12}$$

The explicit solution of the constraint (5.11) is a covariantisation of that described in the previous section. It is given, e.g., in [41].

The action (5.10) is invariant under a second supersymmetry given by

$$\delta_\epsilon X = 2\epsilon^\alpha W_\alpha \, , \tag{5.13}$$

with the parameter $\epsilon_\alpha$ being constrained as in (5.9). Of course, this transformation should be induced by that of $W_\alpha$. The correct supersymmetry transformation of $W_\alpha$ proves to be

$$\delta_\epsilon W_\alpha = \epsilon_\alpha + \frac{1}{4} \epsilon_\alpha \bar{\mathcal{D}}^2 \bar{X} + \mathrm{i} \bar{\epsilon}^{\dot\beta} \mathcal{D}_{\alpha\dot\beta} X - \bar{\epsilon}^{\dot\beta} G_{\alpha\dot\beta} X \ . \tag{5.14}$$

It has the correct flat superspace limit [2], compare with (4.14), and respects the Bianchi identity (5.12),

$$\mathcal{D}^\alpha \delta_\epsilon W_\alpha = \bar{\mathcal{D}}_{\dot\alpha} \delta_\epsilon \bar{W}^{\dot\alpha} \ . \tag{5.15}$$

The dynamical system defined by eqs. (5.10) and (5.11) describes the Maxwell-Goldstone multiplet action for partial $\mathcal{N}=2 \to \mathcal{N}=1$ supersymmetry breaking in those curved spacetimes which are supported by the superspace geometry (5.8), including $\mathbb{R} \times S^3$, $\mathrm{AdS}_3 \times S^1$ and its covering $\mathrm{AdS}_3 \times \mathbb{R}$.

[9] $\mathcal{N}=1$ supersymmetric theories on $\mathbb{R} \times S^3$ were studied in the mid-1980s by Sen [38]. At the component level, the maximally $\mathcal{N}=1$ supersymmetric backgrounds in four dimensions were classified by Festuccia and Seiberg [39]. Their results were re-derived in [40] using the superspace formalism developed in the mid-1990s [34].

## 6 Concluding comments

There are five types of maximally supersymmetric backgrounds in four-dimensional $\mathcal{N}=1$ off-shell supergravity, two of which are well known: Minkowski superspace $\mathbb{R}^{4|4}$ [42, 43] and anti-de Sitter superspace $\mathrm{AdS}^{4|4}$ [44–46]. The remaining three superspaces, $\mathbb{M}^{4|4}_T$, $\mathbb{M}^{4|4}_S$ and $\mathbb{M}^{4|4}_N$, are described by the geometry (5.8) with different choices of $G_a$. All five $\mathcal{N}=1$ superspaces possess $\mathcal{N}=2$ extensions. The Maxwell-Goldstone multiplet on $\mathbb{R}^{4|4}$ for partially broken $\mathcal{N}=2$ Poincaré supersymmetry was found long ago [2, 5]. In this paper, we have constructed the Maxwell-Goldstone multiplets which are defined on $\mathbb{M}^{4|4}_T$, $\mathbb{M}^{4|4}_S$ and $\mathbb{M}^{4|4}_N$ and describe partial $\mathcal{N}=2\to\mathcal{N}=1$ supersymmetry breaking.

In appendix C we demonstrate that no Maxwell-Goldstone multiplet *action* for partial $\mathcal{N}=2\to\mathcal{N}=1$ supersymmetry breaking exists in the case of the anti-de Sitter (AdS) supersymmetry. The reason for this obstruction is the fact that every covariantly constant $\mathsf{SU}(2)$ triplet $G^{ij}_+$ must be proportional to the torsion tensor $S^{ij}$, which is real and covariantly constant in $\mathrm{AdS}^{4|8}$ [47]. As a consequence, the conditions (3.2) and (3.3) are not compatible in $\mathrm{AdS}^{4|8}$. Since the $\mathcal{N}=1$ AdS superspace $\mathrm{AdS}^{4|4}$ is naturally embedded in $\mathrm{AdS}^{4|8}$ as a subspace [77], applying the formalism of section 2 to the case of $\mathrm{AdS}^{4|8}$ allows us to derive a Maxwell-Goldstone multiplet for partially broken $\mathcal{N}=2$ AdS supersymmetry. The corresponding technical details are spelled out in appendix C. However, since the conditions (3.2) and (3.3) are not compatible in $\mathrm{AdS}^{4|8}$, we cannot use this Maxwell-Goldstone multiplet to construct a supersymmetric invariant action.

There exists a one-parameter family of $\mathcal{N}=1$ supersymmetric extensions of the Born-Infeld actions [4]. A unique extension is fixed by the requirement that the action should describe the Maxwell-Goldstone multiplet on $\mathbb{R}^{4|4}$ for partially broken $\mathcal{N}=2$ Poincaré supersymmetry [2, 5]. The same extension is uniquely fixed by the requirement of $\mathsf{U}(1)$ duality invariance [48, 49], which implies the self-duality under superfield Legendre transform discovered by Bagger and Galperin [2]. A curved-superspace extension of the $\mathcal{N}=1$ supersymmetric Born-Infeld action is not unique. However, a unique extension is fixed by the requirement of $\mathsf{U}(1)$ duality invariance [41]. It is given by the action (5.10) in which $X$ is a unique solution to the constraint

$$X+\frac{1}{4}X(\bar{\mathcal{D}}^2-4R)\bar{X}=W^2\,, \tag{6.1}$$

with $R$ the chiral scalar torsion superfield. This action was first proposed in [50]. In the case of anti-de Sitter superspace $\mathrm{AdS}^{4|4}$, the only non-zero components of the superspace torsion are $R$ and $\bar{R}$, which are constant. The corresponding $\mathcal{N}=1$ supersymmetric Born-Infeld action possesses $\mathsf{U}(1)$ duality invariance, however it is not invariant under a second nonlinearly realised supersymmetry, as demonstrated in appendix C. Therefore, this action is not suitable to describe a partial breaking of the $\mathcal{N}=2$ AdS supersymmetry.

In addition to the Maxwell-Goldstone multiplet of [2, 5], there exist other multiplets for partially broken $\mathcal{N}=2$ Poincaré supersymmetry [5, 51, 52]. We believe these models can be generalised to the superspaces $\mathbb{M}^{4|4}_T$, $\mathbb{M}^{4|4}_S$ and $\mathbb{M}^{4|4}_N$ to describe partial $\mathcal{N}=2\to\mathcal{N}=1$ supersymmetry breaking. It would also be interesting to investigate whether some of these models can be extended to describe partially broken $\mathcal{N}=2$ AdS supersymmetry.



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Recently, there has been much interest in models for spontaneously broken local $\mathcal{N}=1$ supersymmetry [53–61], which are based on the use of the nilpotent chiral Goldstino superfield proposed in [62, 63]. Other nilpotent Goldstino superfields can be used to describe spontaneously broken $\mathcal{N}=1$ supergravity [64–66] (for an alternative approach to de Sitter supergravity, see [67]). At the moment it is not clear whether the nilpotent $\mathcal{N}=2$ chiral superfield advocated in the present paper is suitable for the description of partial supersymmetry breaking in $\mathcal{N}=2$ supergravity. It is certainly of interest to develop a superspace description for the models for spontaneous $\mathcal{N}=2 \to \mathcal{N}=1$ local supersymmetry breaking pioneered in [68–70] and further developed, e.g., in [71, 72].


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## A $\mathcal{N}=1$ superspaces over $\mathsf{U}(2)=(S^1\times S^3)/\mathbb{Z}_2$ and $(\mathrm{AdS}_3\times S^1)/\mathbb{Z}_2$

In this appendix we give supermatrix realisations for two maximally supersymmetric backgrounds in 4D $\mathcal{N}=1$ supergravity.

### A.1 $\mathcal{N}=1$ superspace over $\mathsf{U}(2)=(S^1\times S^3)/\mathbb{Z}_2$

Here and in the next appendix, the supergroup $\mathsf{SU}(2|1)$ is defined to consist of complex $(2|1)\times(2|1)$ supermatrices (with $A, D$ bosonic blocks and $B, C$ fermionic ones)

$$g=\left(\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right) \tag{A.1}$$

constrained by

$$g^\dagger \eta g=\eta\,, \qquad \mathrm{Ber}\, g=1\,, \qquad \eta=\left(\begin{array}{c|c} \mathbb{1}_2 & 0 \\ \hline 0 & -1 \end{array}\right)\,. \tag{A.2}$$

We introduce a superspace $\mathcal{M}^{4|4}$ consisting of complex $(2|1)\times(2|0)$ supermatrices (with $h$ bosonic and $\Theta$ fermionic blocks)

$$\mathcal{P}=\begin{pmatrix} h \\ \Theta \end{pmatrix} \tag{A.3}$$

constrained by

$$\mathcal{P}^\dagger\eta\mathcal{P}=\mathbb{1}_2 \quad \Longleftrightarrow \quad h^\dagger h=\mathbb{1}_2+\Theta^\dagger\Theta\,. \tag{A.4}$$

The supermanifold defined by this equation coincides with the 4D $\mathcal{N}=1$ compactified Minkowski superspace (described in detail in section 3 of [73]) on which the superconformal group $\mathsf{SU}(2,2|1)$ acts by well-defined transformations. The bosonic body of the superspace is $\mathsf{U}(2)=(S^1\times S^3)/\mathbb{Z}_2$.



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It is useful to switch from the variables $h$ and $\Theta$ to new ones, $\varphi \in \mathbb{R}$, $u$ and $\theta$, defined as follows:

$$\mathcal{P} = \begin{pmatrix} e^{i\varphi} u \\ \epsilon^{i\varphi}\theta \end{pmatrix}, \qquad u^\dagger u = \mathbb{1}_2 + \theta^\dagger\theta\,, \qquad \det u = \det u^\dagger = \left(1+\theta\theta^\dagger\right)^{-\frac{1}{2}}\,. \tag{A.5}$$

We can represent

$$u = \hat{u}\sqrt{\mathbb{1}_2 + \theta^\dagger\theta}\,, \qquad \hat{u} \in \mathsf{SU}(2)\,. \tag{A.6}$$

The supermatrix (A.5) is invariant under the $\mathbb{Z}_2$ transformation $\varphi \to \varphi + \pi$, $\hat{u} \to -\hat{u}$ and $\theta \to -\theta$. This is the origin of $\mathbb{Z}_2$ in $\mathsf{U}(2) = (S^1 \times S^3)/\mathbb{Z}_2$.

It turns out that the superspace $\mathcal{M}^{4|4}$ introduced above can be identified with the group manifold $\mathsf{SU}(2|1)$. Indeed, it may be checked that every element $g \in \mathsf{SU}(2|1)$ has the form (compare with a similar result in [74])

$$g = \left(\begin{array}{c|c} e^{i\varphi}u & e^{2i\varphi}\left(1+\theta\theta^\dagger\right)^{-\frac{1}{2}} u\theta^\dagger \\ \hline \epsilon^{i\varphi}\theta & e^{2i\varphi}(1+\theta\theta^\dagger)^{\frac{1}{2}} \end{array}\right), \tag{A.7}$$

where $u$ is constrained as in (A.5).

The isometry group of $\mathcal{M}^{4|4}$ is $\mathsf{SU}(2|1) \times \mathsf{U}(2)$. It acts on $\mathcal{M}^{4|4}$ as follows:

$$\mathcal{P} \;\to\; g_{\rm L}\mathcal{P}g_{\rm R}^{-1}\,, \qquad g_{\rm L} \in \mathsf{SU}(2|1)\,, \qquad g_{\rm R} \in \mathsf{U}(2)\,. \tag{A.8}$$

These transformations are holomorphic in terms of the variables $h$ and $\Theta$ (hence the isometry transformations act on a chiral subspace of the full superspace). The isometry group has two $\mathsf{U}(1)$ subgroups that describe $R$-symmetry transformations and time translations. One subgroup corresponds to all diagonal supermatrices (A.7) with $u = \mathbb{1}_2$ and $\theta = 0$. The other subgroup is spanned by all diagonal matrices $e^{i\psi}\mathbb{1}_2$ in $\mathsf{U}(2)$.

On the group manifold $\mathsf{SU}(2|1)$, we can define an action of $\mathsf{SU}(2|1) \times \mathsf{SU}(2|1)$ by the standard rule

$$g \;\to g_{\rm L}\, g\, g_{\rm R}^{-1}\,, \qquad g_{\rm L}, g_{\rm R} \in \mathsf{SU}(2|1)\,. \tag{A.9}$$

These transformations leave invariant the supermetric

$$ds^2 = -\frac{1}{2}\mathrm{Str}\,\mathcal{E}^2\,, \qquad \mathcal{E} = g^{-1}dg\,. \tag{A.10}$$

However, such transformations map the chiral subspace (A.3) to itself only if $g_{\rm R} \in \mathsf{U}(2)$.

### A.2 $\mathcal{N} = 1$ superspace over $\mathsf{U}(1,1) = (\mathrm{AdS}_3 \times S^1)/\mathbb{Z}_2$

We define the supergroup $\mathsf{SU}(1,1|1)$ to consist of complex $(2|1)\times(2|1)$ supermatrices (with $A, D$ bosonic blocks and $B, C$ fermionic ones)

$$g = \left(\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right) \tag{A.11}$$

constrained by

$$g^\dagger \eta g = \eta\,, \qquad \mathrm{Ber}\, g = 1\,, \qquad \eta = \left(\begin{array}{c|c} \sigma_3 & 0 \\ \hline 0 & -1 \end{array}\right). \tag{A.12}$$

Every element $g \in \mathsf{SU}(1,1|1)$ can be written in the form

$$g = \left(\begin{array}{c|c} \mathrm{e}^{\mathrm{i}\varphi} u & \mathrm{e}^{2\mathrm{i}\varphi}\big(1+\theta\sigma_3\theta^\dagger\big)^{-\frac{1}{2}} u\theta^\dagger \\ \hline \epsilon^{\mathrm{i}\varphi}\theta & \mathrm{e}^{2\mathrm{i}\varphi}(1+\theta\sigma_3\theta^\dagger)^{\frac{1}{2}} \end{array}\right) , \tag{A.13}$$

where $u$ is constrained by

$$u^\dagger\sigma_3 u = \sigma_3 + \theta^\dagger\theta \,, \qquad \det u = \det u^\dagger = \big(1+\theta\sigma_3\theta^\dagger\big)^{-\frac{1}{2}} \,. \tag{A.14}$$

We can represent

$$u = \hat{u}\sqrt{\mathbb{1}_2 + \sigma_3\theta^\dagger\theta}\,, \qquad \hat{u} \in \mathsf{SU}(1,1) \,. \tag{A.15}$$

The supermatrix defined by eqs. (A.13) and (A.15) is invariant under the discrete transformation $\varphi \to \varphi + \pi$, $\hat{u} \to -\hat{u}$ and $\theta \to -\theta$.

We introduce a four-dimensional superspace $\widetilde{\mathcal{M}}^{4|4}$ consisting of complex $(2|1)\times(2|0)$ supermatrices (with $h$ and $\Theta$ being bosonic and fermionic blocks, respectively)

$$\mathcal{P} = \begin{pmatrix} h \\ \Theta \end{pmatrix} \equiv \begin{pmatrix} \mathrm{e}^{\mathrm{i}\varphi} u \\ \mathrm{e}^{\mathrm{i}\varphi}\theta \end{pmatrix} , \tag{A.16}$$

where $\varphi$, $u$ and $\theta$ are defined as in (A.13). This superspace can be identified with the group manifold $\mathsf{SU}(1,1|1)$. Its bosonic body is $\mathsf{U}(1,1) = (\mathrm{AdS}_3 \times S^1)/\mathbb{Z}_2$.

The isometry group of $\widetilde{\mathcal{M}}^{4|4}$ is $\mathsf{SU}(1,1|1)\times\mathsf{U}(2)$. It acts on $\widetilde{\mathcal{M}}^{4|4}$ as follows:

$$\mathcal{P} \;\to\; g_{\rm L}\mathcal{P}g_{\rm R}^{-1}\,, \qquad g_{\rm L} \in \mathsf{SU}(1,1|1)\,, \qquad g_{\rm R} \in \mathsf{U}(2) \,. \tag{A.17}$$

These transformations are holomorphic in terms of the variables $h$ and $\Theta$ (hence the isometry transformations acts on a chiral subspace of the full superspace), and leave invariant the supermetric

$$\mathrm{d}s^2 = \frac{1}{2}\mathrm{Str}\,\mathcal{E}^2\,, \qquad \mathcal{E} = g^{-1}\mathrm{d}g \,. \tag{A.18}$$

Unlike the superspace considered in the previous subsection, the dimension parametrised by $\varphi$ is now space-like.

Let us consider the coset space

$$\mathrm{AdS}_{(3|2,0)} := \mathsf{SU}(1,1|1)/\mathsf{U}(1)\,, \tag{A.19}$$

where the subgroup $\mathsf{U}(1)$ of $\mathsf{SU}(1,1|1)$ consists of all diagonal supermatrices (A.13) with $u = \mathbb{1}_2$ and $\theta = 0$. This coset space may be seen to coincide with the 3D (2,0) anti-de Sitter superspace [75]. We recall that in three dimensions, $\mathcal{N}$-extended anti-de Sitter (AdS) superspace exists in several incarnations known as $(p,q)$ AdS superspaces, where the non-negative integers $p \geq q$ are such that $\mathcal{N} = p+q$. The conformally flat $(p,q)$ AdS superspace is

$$\mathrm{AdS}_{(3|p,q)} = \frac{\mathsf{OSp}(p|2;\mathbb{R})\times\mathsf{OSp}(q|2;\mathbb{R})}{\mathsf{SL}(2,\mathbb{R})\times\mathsf{SO}(p)\times\mathsf{SO}(q)} \,. \tag{A.20}$$

In the case $p = \mathcal{N} \geq 4$ and $q = 0$, non-conformally flat AdS superspaces also exist [76].

## B $\mathcal{N} = 2$ superspace over $\mathsf{U}(2) = (S^1 \times S^3)/\mathbb{Z}_2$

$\mathcal{N} = 2$ superspace $\mathcal{M}^{4|8}$ over $\mathsf{U}(2) = (S^1 \times S^3)/\mathbb{Z}_2$ can be realised as the quotient space

$$\mathcal{M}^{4|8} := \mathcal{M}_{\rm L}^{4|4} \times \mathcal{M}_{\rm R}^{4|4} \Big/ \sim \,, \tag{B.1}$$

where $\mathcal{M}_{\rm L}^{4|4}$ and $\mathcal{M}_{\rm R}^{4|4}$ denote two copies of $\mathcal{M}^{4|4}$. The equivalence relation is defined by the rule: two pairs $\boldsymbol{\mathcal{P}} = (\mathcal{P}_{\rm L}, \mathcal{P}_{\rm R})$ and $\boldsymbol{\mathcal{P}}' = (\mathcal{P}'_{\rm L}, \mathcal{P}'_{\rm R})$ are equivalent, $\boldsymbol{\mathcal{P}} \sim \boldsymbol{\mathcal{P}}'$, if

$$\mathcal{P}'_{\rm L} = \mathcal{P}_{\rm L} h \,, \qquad \mathcal{P}'_{\rm R} = \mathcal{P}_{\rm R} h \,, \tag{B.2}$$

for some group element $h \in \mathsf{U}(2)$.

The isometry group of $\mathcal{M}^{4|8}$ is

$$\boldsymbol{G} := G_{\rm L} \times G_{\rm R} \times \mathsf{U}(1) = \mathsf{SU}(2|1) \times \mathsf{SU}(2|1) \times \mathsf{U}(1) \,. \tag{B.3}$$

Given a group element $\boldsymbol{g} = g_{\rm L} \times g_{\rm R} \times {\rm e}^{{\rm i}\psi} \in \boldsymbol{G}$, with $\psi \in \mathbb{R}$, it acts on the pair $\boldsymbol{\mathcal{P}} = (\mathcal{P}_{\rm L}, \mathcal{P}_{\rm R})$ by the rule:

$$(\mathcal{P}_{\rm L}, \mathcal{P}_{\rm R}) \;\to\; (\boldsymbol{g}\mathcal{P}_{\rm L}, \boldsymbol{g}\mathcal{P}_{\rm R}) \,, \qquad \boldsymbol{g}\mathcal{P}_{\rm L} = g_{\rm L}\mathcal{P}_{\rm L}{\rm e}^{{\rm i}\psi} \,, \qquad \boldsymbol{g}\mathcal{P}_{\rm R} = g_{\rm R}\mathcal{P}_{\rm R}{\rm e}^{-{\rm i}\psi} \,. \tag{B.4}$$

The equivalence relation allows us to choose $\mathcal{P}_{\rm R}$ in the form:

$$\mathcal{P}_{\rm R} = \begin{pmatrix} \sqrt{\mathbb{1}_2 + \psi^\dagger \psi} \\ \psi \end{pmatrix} \,. \tag{B.5}$$

The above construction can readily be modified in order to describe the $\mathcal{N} = 2$ superspace over $\mathsf{U}(1,1) = ({\rm AdS}_3 \times S^1)/\mathbb{Z}_2$.

## C Example: the anti-de Sitter supersymmetry

In this appendix we show that the formalism of sections 2 and 3 can be used to define a Goldstone-Maxwell multiplet for partially broken 4D $\mathcal{N} = 2$ anti-de Sitter (AdS) supersymmetry with the following properties: (i) it is the standard Maxwell multiplet with respect to the $\mathcal{N} = 1$ AdS supersymmetry; (ii) it transforms nonlinearly under the second AdS supersymmetry. However, making use of this multiplet does not allow one to construct an invariant action describing the partial $\mathcal{N} = 2 \to \mathcal{N} = 1$ AdS supersymmetry breaking.

To start with, we recall a few definitions concerning the 4D $\mathcal{N} = 2$ AdS superspace

$$\mathrm{AdS}^{4|8} := \frac{\mathsf{OSp}(2|4)}{\mathsf{SO}(3,1) \times \mathsf{SO}(2)} \,,$$

which is a maximally symmetric geometry that originates within the off-shell formulation for $\mathcal{N} = 2$ conformal supergravity developed in [47]. For comprehensive studies of $\mathcal{N} = 2$ supersymmetric field theories in $\mathrm{AdS}_4$, the reader is referred to [77, 78].



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We assume that $\mathrm{AdS}^{4|8}$ is parametrised by local bosonic $(x)$ and fermionic $(\theta,\bar\theta)$ coordinates $z^{\mathcal{M}}=(x^m,\theta^\mu_\imath,\bar\theta^\imath_{\dot\mu})$ (where $m=0,1,2,3$, $\mu=1,2$, $\dot\mu=1,2$ and $\imath=\underline{1},\underline{2}$). The corresponding covariant derivatives

$$\mathcal{D}_{\mathcal{A}}=(\mathcal{D}_a,\mathcal{D}^i_\alpha,\bar{\mathcal{D}}^{\dot\alpha}_i)=E_{\mathcal{A}}{}^{\mathcal{M}}\partial_{\mathcal{M}}+\frac{1}{2}\Omega_{\mathcal{A}}{}^{bc}M_{bc}+\Phi_{\mathcal{A}}{}^{ij}J_{ij}\,,\qquad i,j=\underline{1},\underline{2} \tag{C.1}$$

obey the algebra [77]

$$\{\mathcal{D}^i_\alpha,\mathcal{D}^j_\beta\}=4S^{ij}M_{\alpha\beta}+2\varepsilon_{\alpha\beta}\varepsilon^{ij}S^{kl}J_{kl}\,,\qquad \{\mathcal{D}^i_\alpha,\bar{\mathcal{D}}^{\dot\beta}_j\}=-2\mathrm{i}\delta^i_j(\sigma^c)_\alpha{}^{\dot\beta}\mathcal{D}_c\,, \tag{C.2a}$$

$$[\mathcal{D}_a,\mathcal{D}^j_\beta]=\frac{\mathrm{i}}{2}(\sigma_a)_{\beta\dot\gamma}S^{jk}\bar{\mathcal{D}}^{\dot\gamma}_k\,,\qquad [\mathcal{D}_a,\bar{\mathcal{D}}^{\dot\beta}_j]=\frac{\mathrm{i}}{2}(\tilde\sigma_a)^{\dot\beta\gamma}S_{jk}\mathcal{D}^k_\gamma\,, \tag{C.2b}$$

$$[\mathcal{D}_a,\mathcal{D}_b]=-S^2M_{ab}\;. \tag{C.2c}$$

The $\mathsf{SU}(2)$ triplet $S^{ij}$ is the only non-vanishing component of the superspace torsion in $\mathrm{AdS}^{4|8}$; it is *covariantly constant* and real

$$\mathcal{D}_{\mathcal{A}}S^{ij}=0\,,\qquad \bar S^{ij}=S^{ij}\;. \tag{C.3}$$

The parameter $S^2:=\frac{1}{2}S^{ij}S_{ij}=\text{const}$ is positive, and therefore (C.2c) gives the algebra of covariant derivatives in $\mathrm{AdS}_4$.

The isometry transformations of $\mathrm{AdS}^{4|8}$ form the supergroup $\mathsf{OSp}(2|4)$. In the infinitesimal case, an isometry transformation is described by a Killing supervector field $\xi^{\mathcal{A}}E_{\mathcal{A}}$, with $E_{\mathcal{A}}=E_{\mathcal{A}}{}^{\mathcal{M}}\partial_{\mathcal{M}}$, defined to obey the equation

$$\Big[\xi+\frac{1}{2}l^{bc}M_{bc}+\rho S^{jk}J_{jk},\mathcal{D}_{\mathcal{A}}\Big]=0\,,\qquad \xi:=\xi^{\mathcal{B}}\mathcal{D}_{\mathcal{B}}=\xi^b\mathcal{D}_b+\xi^\beta_j\mathcal{D}^j_\beta+\bar\xi^i_{\dot\beta}\bar{\mathcal{D}}^{\dot\beta}_j\,, \tag{C.4}$$

for some real antisymmetric tensor $l^{bc}(z)$ and scalar $\rho(z)$ parameters. It turns out that the Killing equation (C.4) uniquely determines the parameters $\xi^\alpha_i$, $l^{cd}$ and $\rho$ in terms of $\xi^a$. A similar property exists for superspace isometry transformations in any number of dimensions [35]. The specific feature of the 4D $\mathcal{N}=2$ AdS superspace is that the parameters $\xi^{\mathcal{A}}$ and $l^{ab}$ are uniquely expressed in terms of $\rho$ [77].

Due to (C.2), the $\mathsf{SU}(2)$ gauge freedom can be used to choose the $\mathsf{SU}(2)$ connection $\Phi_{\mathcal{A}}{}^{ij}$ in (C.1) to look like $\Phi_{\mathcal{A}}{}^{ij}=\Phi_{\mathcal{A}}S^{ij}$, for some one-form $\Phi_{\mathcal{A}}$ describing the residual $\mathsf{U}(1)$ connection associated with the generator $S^{ij}J_{ij}$. Then $S^{ij}$ becomes a constant iso-triplet, $S^{ij}=\text{const}$. The remaining global $\mathsf{SU}(2)$ rotations can take $S^{ij}$ to any position on the two-sphere of radius $S$. We make the choice

$$S^{\underline{12}}=0\,,\qquad \mu:=-S^{\underline{22}}\,,\qquad \bar\mu=-S^{\underline{11}}\,, \tag{C.5}$$

with $|\mu|=S$. This choice must be used in order to embed an $\mathcal{N}=1$ AdS superspace, $\mathrm{AdS}^{4|4}$, into the full $\mathcal{N}=2$ AdS superspace [77].

As already mentioned, the choice $S^{\underline{12}}=0$ is required for embedding $\mathrm{AdS}^{4|4}$ into $\mathrm{AdS}^{4|8}$. By applying certain general coordinate and local $\mathsf{U}(1)$ transformations in $\mathrm{AdS}^{4|8}$, it is possible to identify $\mathrm{AdS}^{4|4}$ with the surface $\theta^\mu_{\underline{2}}=0$ and $\bar\theta^{\underline{2}}_{\dot\mu}=0$. The covariant derivatives for $\mathrm{AdS}^{4|4}$,

$$D_A=(\mathcal{D}_a,\mathcal{D}_\alpha,\bar{\mathcal{D}}^{\dot\alpha})=E_A{}^M\partial_M+\frac{1}{2}\Omega_A{}^{bc}M_{bc}\,, \tag{C.6}$$

are related to (C.1) as follows

$$\mathcal{D}_\alpha := \mathcal{D}^{\underline{1}}_\alpha\big| , \qquad \bar{\mathcal{D}}^{\dot\alpha} := \bar{\mathcal{D}}^{\dot\alpha}_{\underline{1}}\big| , \tag{C.7}$$

and similarly for the vector covariant derivative. Here the bar-projection is defined by

$$U| := U(x,\theta_\imath,\bar\theta^\imath)|_{\theta_{\underline{2}}=\bar\theta^{\underline{2}}=0} , \tag{C.8}$$

for any $\mathcal{N}=2$ tensor superfield $U(x,\theta_\imath,\bar\theta^\imath)$. It follows from (C.2) that the $\mathcal{N}=1$ covariant derivatives obey the algebra

$$\{\mathcal{D}_\alpha,\mathcal{D}_\beta\} = -4\bar\mu M_{\alpha\beta} , \qquad \{\bar{\mathcal{D}}_{\dot\alpha},\bar{\mathcal{D}}_{\dot\beta}\} = 4\mu \bar M_{\dot\alpha\dot\beta} , \qquad \{\mathcal{D}_\alpha,\bar{\mathcal{D}}_{\dot\beta}\} = -2\mathrm{i}\mathcal{D}_{\alpha\dot\beta} , \tag{C.9a}$$

$$[\mathcal{D}_a,\mathcal{D}_\beta] = -\frac{\mathrm{i}}{2}\bar\mu(\sigma_a)_{\beta\dot\gamma}\bar{\mathcal{D}}^{\dot\gamma} , \qquad [\mathcal{D}_a,\bar{\mathcal{D}}_{\dot\beta}] = \frac{\mathrm{i}}{2}\mu(\sigma_a)_{\gamma\dot\beta}\mathcal{D}^{\gamma} , \tag{C.9b}$$

$$[\mathcal{D}_a,\mathcal{D}_b] = -|\mu|^2 M_{ab} , \tag{C.9c}$$

which indeed corresponds to the $\mathcal{N}=1$ AdS superspace (see [34] for more details). As a result, every $\mathcal{N}=2$ supersymmetric field theory in AdS$^{4|8}$ can be reformulated as some theory in AdS$^{4|4}$.

Given an $\mathcal{N}=2$ tensor superfield $U(x,\theta_\imath,\bar\theta^\imath)$, its infinitesimal $\mathsf{OSp}(2|4)$ transformation law is

$$\delta_\xi U = -\Big(\xi + \frac12 l^{bc}M_{bc} + \rho S^{jk}J_{jk}\Big)U . \tag{C.10}$$

Upon reduction to AdS$^{4|4}$, this transformation law turns into a superposition of several independent $\mathcal{N}=1$ transformations. Evaluating the bar-projection of $\xi$ gives

$$\xi| = \lambda + \varepsilon^\alpha \mathcal{D}^{\underline{2}}_\alpha| + \bar\varepsilon_{\dot\alpha}\bar{\mathcal{D}}^{\dot\alpha}_{\underline{2}}| , \qquad \lambda = \lambda^A\mathcal{D}_A = \lambda^a\mathcal{D}_a + \lambda^\alpha\mathcal{D}_\alpha + \bar\lambda_{\dot\alpha}\bar{\mathcal{D}}^{\dot\alpha} , \tag{C.11a}$$

where we have introduced

$$\lambda^a := \xi^a| , \quad \lambda^\alpha := \xi^\alpha_{\underline{1}}| , \quad \bar\lambda_{\dot\alpha} := \bar\xi^{\underline{1}}_{\dot\alpha}| , \qquad \varepsilon^\alpha := \xi^\alpha_{\underline{2}}| , \quad \bar\varepsilon_{\dot\alpha} := \xi^{\underline{2}}_{\dot\alpha}| . \tag{C.11b}$$

We denote the bar-projection of the parameters $l_{ab}$ and $\rho$ as

$$\omega_{ab} := l_{ab}| , \qquad \varepsilon := \rho| . \tag{C.12}$$

It holds that

$$\omega_{\alpha\beta} = \mathcal{D}_\alpha\lambda_\beta = \mathcal{D}_\beta\lambda_\alpha . \tag{C.13}$$

Now, the bar-projection of (C.10) takes the form

$$\delta_\xi U| = -\Big(\lambda + \frac12\omega^{ab}M_{ab}\Big)U| - \Big(\varepsilon^\alpha(\mathcal{D}^{\underline{2}}_\alpha U)| + \bar\varepsilon_{\dot\alpha}(\bar{\mathcal{D}}^{\dot\alpha}_{\underline{2}}U)|\Big) + \varepsilon(\bar\mu J_{\underline{11}} + \mu J_{\underline{22}})U| . \tag{C.14}$$

The first term on the right is an infinitesimal $\mathsf{OSp}(1|4)$ transformation generated by $\lambda$. The parameters $\lambda$ and $\omega^{bc}$ obey the equation

$$\Big[\lambda + \frac12\omega^{bc}M_{bc}, \mathcal{D}_A\Big] = 0 , \tag{C.15}$$

which defines the Killing supervector field of $\mathrm{AdS}^{4|4}$ [34]. The second and third terms on the right of (C.14) prove to describe the second supersymmetry and $\mathsf{U}(1)$ transformations. The corresponding parameters $\varepsilon_\alpha$, $\bar{\varepsilon}_{\dot{\alpha}}$ and $\varepsilon$ have the properties

$$\varepsilon_\alpha = \frac{1}{2}\mathcal{D}_\alpha \varepsilon\,, \qquad \mathcal{D}_\alpha \bar{\mathcal{D}}_{\dot{\alpha}} \varepsilon = 0\,, \qquad \big(\mathcal{D}^2 - 4\bar{\mu}\big)\varepsilon = 0\,. \tag{C.16}$$

The parameter $\varepsilon$ was originally introduced in [79, 80].

We are now prepared to analyse the nilpotent $\mathcal{N}=2$ chiral superfield $\mathcal{Z}$ constrained by (2.1) in the case that the background superspace is $\mathrm{AdS}^{4|8}$. We recall that a necessary ingredient of the construction described in section 3 is that $G^{ij}$ is covariantly constant, $\mathcal{D}_{\mathcal{A}} G^{ij} = 0$. We require this condition to hold in $\mathrm{AdS}^{4|8}$, which implies that $G^{ij}$ is proportional to $S^{ij}$

$$G^{ij} = \kappa S^{ij}\,, \tag{C.17}$$

where $\kappa$ is a real constant. In accordance with (C.5), we have $G^{\underline{12}} = 0$. The parameter $\kappa$ can be chosen to have any given non-zero value by means of rescaling the chiral superfield $\mathcal{Z}$. We choose $\kappa = |\mu|$, and hence $G^{\underline{11}} = -|\mu|\bar{\mu}$ and $G^{\underline{22}} = -|\mu|\mu$.

The degrees of freedom described by $\mathcal{Z}$ are those of an Abelian $\mathcal{N}=1$ vector multiplet in $\mathrm{AdS}^{4|4}$. Indeed, upon reduction to the $\mathcal{N}=1$ AdS superspace, the $\mathcal{N}=2$ chiral scalar $\mathcal{Z}$ leads to two chiral superfields, $X$ and $W_\alpha$, defined as

$$X := \mathcal{Z}|\,, \qquad \bar{\mathcal{D}}_{\dot{\alpha}} X = 0\,, \tag{C.18a}$$

$$W_\alpha := -\frac{\mathrm{i}}{2}\mathcal{D}^{\underline{2}}_\alpha \mathcal{Z}|\,, \qquad \bar{\mathcal{D}}_{\dot{\alpha}} W_\alpha = 0\,. \tag{C.18b}$$

One may check that the $\mathcal{N}=2$ constraints (2.1) imply the Bianchi identity

$$\mathcal{D}^\alpha W_\alpha = \bar{\mathcal{D}}_{\dot{\alpha}} \bar{W}^{\dot{\alpha}}\,, \tag{C.19a}$$

as well as the nonlinear constraint

$$-\,\mathrm{i}\mu|\mu|X + \frac{1}{4}X\big(\bar{\mathcal{D}}^2 - 4\mu\big)\bar{X} = W^2\,, \qquad W^2 := W^\alpha W_\alpha\,. \tag{C.19b}$$

Eq. (C.19a) tells us that $W_\alpha$ is the chiral field strength of a Maxwell multiplet in $\mathrm{AdS}^{4|4}$. Eq. (C.19b) is of the same type as the constraint (6.1), which generates the $\mathcal{N}=1$ locally supersymmetric Born-Infeld action with $\mathsf{U}(1)$ duality invariance. The constraint (C.19b) is uniquely solved by expressing $X$ in terms of $W^2$ and $\bar{W}^2$ and their covariant derivatives, in complete analogy with the general supergravity analysis of [41].

In accordance with (C.10), the infinitesimal $\mathsf{OSp}(2|4)$ transformation of $\mathcal{Z}$ is $\delta\mathcal{Z} = -\xi\mathcal{Z}$. Using this result, it is straightforward to derive the transformation laws of $X$ and $W_\alpha$ under the second supersymmetry and $\mathsf{U}(1)$ transformations described by the superfield parameter $\varepsilon$. Making use of the constraints obeyed by $\mathcal{Z}$ and $X$, we obtain

$$\delta_\varepsilon X = -2\mathrm{i}\varepsilon^\alpha W_\alpha\,, \tag{C.20a}$$

$$\delta_\varepsilon W_\alpha = \mathrm{i}\varepsilon_\alpha\Big[\mathrm{i}\mu|\mu| - \frac{1}{4}\big(\bar{\mathcal{D}}^2 - 4\mu\big)\bar{X} - \mu X\Big] - \bar{\varepsilon}^{\dot{\beta}}\mathcal{D}_{\alpha\dot{\beta}}X + \frac{\mathrm{i}}{2}\mu\varepsilon\mathcal{D}_\alpha X\,. \tag{C.20b}$$



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One can check that $\delta_\varepsilon X$ and $\delta_\varepsilon W_\alpha$ preserve the constraints (C.19). Due to (C.16), the variation $\delta_\varepsilon W_\alpha$ can be rewritten in the form

$$\delta_\varepsilon W_\alpha = -\frac{\mathrm{i}}{8}(\bar{\mathcal{D}}^2 - 4\mu)\Big[2\Big(\bar{X} - X + \mathrm{i}|\mu|\Big)\varepsilon_\alpha - \varepsilon\mathcal{D}_\alpha X\Big]\,, \tag{C.20c}$$

which makes manifest the chirality of $\delta_\varepsilon W_\alpha$. It follows from (C.20) that the second supersymmetry and $\mathsf{U}(1)$ transformations are nonlinearly realised.

Let us consider the supersymmetric and $\mathsf{U}(1)$ duality invariant Born-Infeld action in the $\mathcal{N} = 1$ AdS superspace[10]

$$S = -\frac{\mathrm{i}}{4}|\mu|\mu\int \mathrm{d}^4x\,\mathrm{d}^2\theta\,\mathcal{E}\,X + \text{c.c.}\,, \tag{C.21}$$

with $X$ constrained by (C.19b). The action is manifestly invariant under the isometry transformations of $\mathrm{AdS}^{4|4}$, with the infinitesimal transformation law of $W_\alpha$ being

$$\delta W_\alpha = -\lambda W_\alpha - \omega_\alpha{}^\beta W_\beta\ . \tag{C.22}$$

However, the action is not invariant under the transformation (C.20),

$$\delta_\varepsilon S = 2|\mu|^3\int \mathrm{d}^4x\,\mathrm{d}^2\theta\mathrm{d}^2\bar{\theta}\,E\,\varepsilon V\,, \tag{C.23}$$

where the real scalar $V$ denotes the unconstrained prepotential of the vector multiplet,

$$W_\alpha = -\frac{1}{4}(\bar{\mathcal{D}}^2 - 4\mu)\mathcal{D}_\alpha V\ . \tag{C.24}$$

Eq. (C.23) is a unique feature that distinguishes $\mathrm{AdS}_4$ from the other maximally supersymmetric backgrounds we have studied in this paper.



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